PSA Search Simulation
Looking For A Lost Drill

B Background

Scientists at NASA are building a robot called the Personal Satellite Assistant, or PSA. The PSA's mission is to keep the astronauts safe and to assist them with their chores on space-based vehicles such as the International Space Station (ISS), a Crew Exploration Vehicle, or even on Mars. This small round robot will float in microgravity and move autonomously (without direction from people). It will keep track of the astronauts' schedules, monitor supplies, assist with scientific experiments, communicate with Mission Control, and help keep the astronauts safe by monitoring the air composition and temperature.

NASA engineers have created a model of the PSA with a diameter of 30.5 centimeters (12 inches). The engineers' goal is to build a PSA with a 20-centimeter (8-inch) diameter, because it will be safer and will require less power to move around.

I Introduction

In this lesson, students use calculations, visualization, and logical reasoning to discover that the surface area and volume of a sphere changes disproportionately when its radius changes. They will use ratios to compare surface areas and volumes of different sports balls to determine how the radius of a sphere affects these parameters. Students will also compare sports balls to determine which is closest in size to the PSA.

M Main Concept

The surface area and volume of a sphere change disproportionately with radius.

N NASA Relevance

NASA scientists and engineers working on the PSA project need to reduce the volume and mass of the PSA because of the high cost of transportation for leaving our home planet, limited space in space-based vehicles, and for safety reasons. Even though they use computer programs to design the PSA and its components, a basic understanding of volume and surface area is essential in order to design the PSA and the shape of the components that go inside it.

P Prerequisite Skills

Students should be able to:
- Calculate the surface area and volume of a sphere.
- Conduct basic math operations using decimals.
- Calculate and interpret ratios.

I Instructional Objectives

During this lesson, students will:
1. Explain that the surface area and volume of a sphere change disproportionately when the radius changes.

NATIONAL EDUCATION STANDARDS		
Fully Met	**Partially Met**	**Addressed**
NCTM (6-8) *Geometry #1* *Measurement (#2* **2061: 9B (6-8) #1** **2061: 9C (6-8) #1**	**NCTM (6-8)** *Geometry #4* **2061: 3B (6-8) #1**	**ITEA 8** **NCTM (6-8)** *Measurement #1* **NSES E (5-8) #1** **2061: 2B (6-8) #1**

Ⓜ Major Concepts

- Mathematical statements can be used to describe how one quantity changes when another changes.
- Ratios can be used to see mathematical relationships or to compare changes.
- As the radius of a sphere increases, the volume increases more than the surface area.

Ⓜ Materials and Equipment

- Computer with an Internet* connection
- Pictures of the PSA, obtained from the PSA Web site at http://psa.arc.nasa.gov/. In the Activities section, select PSA Systems to get to http://psa.arc.nasa.gov/syst.shtml.
- 1 baseball, 1 softball, 1 volleyball, and 1 basketball per every two groups of 2-3 students
- 1 30.5-centimeter (12-inch) diameter globe
- Rulers or measuring tape for each group
- Calculators
- 1 copy of the Student Handout sheet for each student or group of students
- String as long as the circumference of the largest ball for each group

S **System Requirements to Run PSA Web Site Activities**

Platform	Browser
Windows 95 Windows 98 Windows Me	Internet Explorer 4.0 or later (Internet Explorer 5.0 or later is recommended), Netscape Navigator 4 or later, Netscape 7.0 or later (Netscape 6 is not recommended) JavaScript enabled
Windows NT Windows 2000 Windows XP or later	Internet Explorer 4.0 or later, Netscape Navigator 4 or later, Netscape 7.0 or later, with standard install defaults (Netscape 6 is not recommended) JavaScript enabled
Macintosh: 8.6 thru 9.2	Netscape 4.5 or later (Netscape Communicator 4.7 or Netscape 7.0 are recommended), Netscape 7.0 or later, (Netscape 6 is not recommended) Microsoft Internet Explorer 5.0 or later JavaScript enabled
Macintosh OS X 10.1 or later	Netscape 7.0 or later (Netscape 6 is not recommended), Microsoft Internet Explorer 5.1 or later JavaScript enabled
Browser plug-ins	Flash Player 6 or higher QuickTime Player 6 or higher

T **Time for Activity**
1 class period

LESSON   Engage

Show students pictures of the Personal Satellite Assistant (PSA). Ask students how the PSA might look different if its surface area were reduced by half. Ask how the function of the PSA might be different if its volume were reduced by half.

Tell students they are going to investigate how the surface area and volume of a sphere change as its radius changes. Explain that they will also determine how big the PSA is in real life. Remind students that NASA engineers have created a 30.5-centimeter (12-inch) diameter model of the PSA, but they want to shrink it to 20 centimeters (8 inches) in diameter.

20 cm diam.

30.5 cm diam.

Show students a 30.5-centimeter (12-inch) diameter globe. Remind students that the globe is roughly the size of the current PSA model.
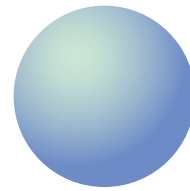
Ask students what information they need to calculate its surface area and volume. Listen to the students' responses. If they appear confused, draw three circles of different sizes and ask students how to calculate the area of each of the circles. The only information they need is the radius of the sphere. Review the properties of a sphere.

Ask students what formulas are necessary to calculate the surface area and volume of the sphere. Write these formulas on the board.

Surface Area = 4 x π x radius x radius
Volume = 4/3 x π x radius x radius x radius

Show students a baseball, softball, volleyball, and basketball. Ask them if they think the surface area and volume of a sphere change at equal rates as the spheres increase from the size of a baseball to the size of a basketball.

Ask students how they will verify their hypotheses.

Explore

Pass out rulers, calculators, and copies of the Student Handout. Make the balls available so each group can examine one at a time. Ask students to measure the radii of the balls and calculate the surface areas and volumes.

> **Note:** *Students can do this by using a string to measure the circumference of the ball, making a circle of this circumference on a flat surface, or measuring the diameter of this circle and taking half of the diameter to calculate radius. Stress to students that this will be an approximation of the length, due to errors that can occur in the process. In fact, you may want to either have students make multiple measurements or compare all students' results for the radius of each ball to verify results before having students calculate volume and surface area.*

Students should record their results in the Student Handout sheet.

Instruct students to calculate the volume-to-surface area ratios for each of the spheres.

Explain

☐ Ask: "Which ball is closest in size to the intended final size of the PSA?"
■ *Answer:*   *The volleyball; its diameter is about 20 centimeters (8 inches).*

☐ Ask: "How does the volume-to-surface area ratio of a sphere changes as its radius gets bigger?"
■ *Answer:*   *Students should be able to recognize that the ratio gets larger as the radius increases.*

Ask students to consider the differences between the volleyball and the basketball.

☐ Ask: "How much bigger was the radius of the basketball than the volleyball?"
■ *Answer:*   *About 1.6 centimeters.*

☐ Ask: "How much more surface area did the basketball have than the volleyball?"
■ *Answer:*   *About 465 square centimeters.*

☐ Ask: "How much more volume did the basketball have than the volleyball?"
■ *Answer:*   *About 2646 cubic centimeters!*

Ask students what conclusions they can draw about the ratio of surface area to volume as the radius changes. They should articulate that as the radius increases, the volume increases more than the surface area.

Extend

Ask students to apply this idea to the PSA. What will happen as the PSA is shrunk from a 30.5-centimeter (12-inch) diameter to a 20-centimeter (8-inch) diameter sphere? What will be the advantages and disadvantages of shrinking the PSA? (At this point, you may want to discuss how engineers must make trade-offs.) Why do they think NASA decided on this size? What would happen if they used an even smaller size?

Evaluate

As a class, create an assessment rubric for this activity. Suggested criteria for the rubric include:
- Accurate and consistent measurements of the volumes and surface areas of spheres.
- Appropriate calculations of volume-to-surface area ratios.
- Correct assessment of the relationship between the volume-to-surface area ratios and radii of spheres.
- Correct determination of which ball most resembles the PSA.
- Correct discussion of the advantages and disadvantages of shrinking the PSA.
- Clear written presentation of results.
- Clear oral presentation of results.

Use the rubric to assess students' interpretation of how size affects the PSA's volume and surface area and ensure they have mastered the major concepts and math skills.

Student Handout

Formulas

> **Surface Area = 4 x π x radius x radius**
> **Volume = 4/3 x π x radius x radius x radius**

Surface Area vs. Volume of a Sphere

1. Fill out the table below.

	Radius (centimeters)	**Surface Area (square centimeters)**	**Volume (cubic centimeters)**	**Ratio of Volume to Surface Area**
Baseball				
Softball				
Volleyball				
Basketball				

2. Which of the above balls is closest in size to the intended final PSA?

_____

3. Describe the relationship between volume-to-surface area ratio and the radius of the sphere:

_____

_____

_____

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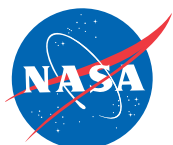
Shrinking the PSA

1. What will happen to the volume and surface area as the PSA is shrunk from a 30.5-centimeter (12-inch) diameter to a 20–centimeter (8-inch) diameter sphere?

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2. What are the advantages and disadvantages of shrinking the PSA?

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3. Why do you think NASA decided on this size for the PSA?

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4. What would happen if they used an even smaller size?

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Answer Key

Formulas

Surface Area = 4 x π x radius x radius
Volume = 4/3 x π x radius x radius x radius

Surface Area vs. Volume of a Sphere

	Radius (centimeters)	Surface Area (square centimeters)	Volume (cubic centimeters)	Ratio of Volume to Surface Area
Baseball	3.56	159.26	188.99	1.19
Softball	4.85	295.59	477.87	1.62
Volleyball	10.54	1396.02	4904.68	3.51
Basketball	12.17	1861.19	7550.23	4.06

Which of the above balls is closest in size to the intended final PSA?
The volleyball is closest in size to the intended final size of the PSA.

Describe the relationship between volume-to-surface area ratio and the radius of the sphere.
As the radius gets bigger, the volume to surface area ratio increases. The volume increases more than the surface area.

Shrinking the PSA

What will happen to the volume and surface area as the PSA is shrunk from a 30.5-centimeter (12-inch) diameter to a 20-centimeter (8-inch) diameter sphere?
As the PSA is shrunk to a 20-centimeter (8-inch) diameter sphere, its volume will decrease much more than its surface area. The ratio of volume to surface area will shrink.

What are the advantages and disadvantages of shrinking the PSA?
The advantages will be that it will be easier to move around, requiring less power and will be less dangerous when it hits something. It will be easier for it to go into small spaces. Also, it will have more surface area per volume to release heat from its components. The disadvantages will be that it will have a smaller volume, so it won't be able to hold as much inside of it.

Why do you think NASA decided on this size for the PSA?
NASA probably decided on this size because there are a lot of advantages to having a small size, but they couldn't go any smaller or it wouldn't be big enough for all the parts it needs.

Sample Scoring Tool

4
- Calculations are correct and clearly presented.
- Students accurately interpret ratios.
- Reasoning is logical and clear explanations are provided.
- Oral and written presentations are clear.

3
- Most calculations are correct and attempts are made to present them clearly.
- Students draw appropriate conclusions from ratios.
- Attempts are made to reason logically and provide clear explanations.
- Attempts are made to provide clear oral and written presentations.

2
- Some calculations are correct and attempts moderately clear.
- Students have difficulty interpreting ratios.
- Explanations demonstrate limited logical bases.
- Oral and written presentation skills need improvement.

1
- Few calculations are correct and attempts are unclear.
- Students do not demonstrate adequate understanding of ratios.
- Explanations do not demonstrate understanding of lesson content.
- Oral and written presentations do not effectively express results or reasoning